Kinematics of Wheeled Robots

https://www.youtube.com/watch?v=giS41utjlbU

Wheeled Mobile Robots

- robot can have one or more wheels that can provide
 - steering (directional control)
 - power (exert a force against the ground)
- an ideal wheel is
 - perfectly round (perimeter $2\pi r$)
 - moves in the direction perpendicular to its axis

Wheel





Deviations from Ideal

This illustration gives a good sense of the steering and throttling you'll have to input to keep your car drifting. When snapping the car from its full drift angle in one direction to full drift in the opposite direction, be prepared for the rear end to come around with more force then when initiating a drift from straight-ahead running. Give yourself plenty of space as you master your technique so you don't slap a curb or something equally immobile!

1 Steer hard into the turn to initiate a slide, then countersteer before the car loops out.

Continue countersteering to maintain the slide. It's a balancing act!



Now you're setting up for the next turn. The goal is to drift all the way through without straightening out for more than a moment as the rear end swings around. 2

4 You'll have burned off some speed by now, so be prepared to pin the throttle to keep the rear wheels slipping. Traction kills drift!

3

Here's where it gets tricky. Keeping the rear end sliding through turns is relatively easy, but drifting down a straight is tough. As you lose momentum, the car will straighten out, but you'll be surprised how long you can hang it out. Good luck!

http://worlddrifting.com/wp-content/uploads/2010/02/drifting-techniques.jpg 4

Instantaneous Center of Curvature

- for smooth rolling motion, all wheels in ground contact must
 - follow a circular path about a common axis of revolution
 - each wheel must be pointing in its correct direction
 - revolve with an angular velocity consistent with the motion of the robot
 - each wheel must revolve at its correct speed

Instantaneous Center of Curvature



(a) 3 wheels with roll axes intersecting at a common point (the instantaneous center of curvature, ICC).
(b) No ICC exists. A robot having wheels shown in
(a) can exhibit smooth rolling motion, whereas a robot with wheel arrangement
(b) cannot.

Castor Wheels

provide support but not steering nor propulsion



Differential Drive

two independently driven wheels mounted on a common axis



Differential Drive

• angular velocity ω about the ICC defines the wheel ground velocities v_r and v_{ℓ} ground speed (m/s) ground speed of left when right when the distance be



distance between ICC and right wheel

 $v_r = \omega(R + \frac{\ell}{2})$ $v_{\ell} = \omega(R - \frac{\ell}{2})$

distance between ICC and left wheel

https://opencurriculum.org/5481/circular-motion-linear-and-angular-speed/

Differential Drive

• given the wheel ground velocities it is easy to solve for the radius, R, and angular velocity ω



 $v_\ell = -v_r$ ____ robot spins in place

Tracked Vehicles

- similar to differential drive but relies on ground slip or skid to change direction
 - kinematics poorly determined by motion of treads



http://en.wikipedia.org/wiki/File:Tucker-Kitten-Variants.jpg



Steered Wheels: Bicycle

- important to remember the assumptions in the kinematic model
 - smooth rolling motion in the plane
- does not capture all possible motions
 - http://www.youtube.com/watch?v=Cj6hoI-G6tw&NR=I#t=0m25s

Mecanum Wheel

a normal wheel with rollers mounted on the circumference



http://blog.makezine.com/archive/2010/04/3d-printable-mecanum-wheel.html

- https://www.youtube.com/watch?v=O7FbDy-gE70
- https://www.youtube.com/watch?v=mUoftURFsxM
- http://ftp.mi.fu-berlin.de/pub/Rojas/omniwheel/Diegel-Badve-Bright-Potgieter-Tlale.pdf

Mecanum Wheel

Direction of	
Movement	Wheel Actuation
Forward	All wheels forward same speed
Reverse	All wheels backward same speed
Right Shift	Wheels 1, 4 forward; 2, 3 backward
Left Shift	Wheels 2, 3 forward; 1, 4 backward
CW Turn	Wheels 1, 3 forward; 2, 4 backward
CCW Turn	Wheels 2, 4 forward; 1, 3 backward

To the right: This is a top view looking down on the drive platform. Wheels in Positions 1, 4 should make X- pattern with Wheels 2, 3. If not set up like shown, wheels will not operate correctly.



AndyMark Mecanum wheel specification sheet http://dlpytrrjwm20z9.cloudfront.net/MecanumWheelSpecSheet.pdf

- serial manipulators
 - given the joint variables, find the pose of the end-effector
- mobile robot
 - given the control variables as a *function of time*, find the pose of the robot
 - for the differential drive the control variables are often taken to be the ground velocities of the left and right wheels
 - it is important to note that the wheel velocities are needed as functions of time; a differential drive that moves forward and then turns right ends up in a very different position than one that turns right then moves forward!

robot with pose [x y θ]^T moving with velocity V in a direction
 θ measured relative the x axis of {W}:



• for a robot starting with pose $[x_0 \ y_0 \ \theta_0]^T$ moving with velocity V(t) in a direction $\theta(t)$:



$$x(t) = x_0 + \int_0^t V(t) \cos(\theta(t)) dt$$

$$y(t) = y_0 + \int_0^t V(t) \sin(\theta(t)) dt$$

$$\theta(t) = \theta_0 + \int_0^t \omega(t) dt$$

$$\zeta$$

argular
welcich about ICC

for differential drive:

$$V = \frac{V_{R} + V_{L}}{2}$$

$$x(t) = x_0 + \frac{1}{2} \int_0^t (v_r(t) + v_\ell(t)) \cos(\theta(t)) dt$$

$$y(t) = y_0 + \frac{1}{2} \int_0^t (v_r(t) + v_\ell(t)) \sin(\theta(t)) dt$$

$$\theta(t) = \theta_0 + \frac{1}{\ell} \int_0^t (v_r(t) - v_\ell(t)) dt$$





Mobile Robot Forward Kinematics

what is the position of the ICC in {W}?





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- assuming smooth rolling motion at each point in time the differential drive is moving in a circular path centered on the ICC
 - thus, for a small interval of time δt the change in pose can be computed as a rotation about the ICC



- computing the rotation about the ICC
 - translate so that the ICC moves to the origin of {W}
 - rotate about the origin of {W}
 - 3. translate back to the original ICC



- computing the rotation about the ICC
 - I. translate so that the ICC moves to the origin of $\{W\}$
 - 2. rotate about the origin of {W}
 - 3. translate back to the original ICC

$$ICC = \begin{bmatrix} x - R\sin\theta \\ y + R\cos\theta \end{bmatrix} = \begin{bmatrix} ICC_x \\ ICC_y \end{bmatrix}$$

$$\begin{bmatrix} x \\ y \end{bmatrix} - \begin{bmatrix} ICC_x \\ ICC_y \end{bmatrix} = \begin{bmatrix} x - ICC_x \\ y - ICC_y \end{bmatrix}$$

- computing the rotation about the ICC
 - 1. translate so that the ICC moves to the origin of $\{W\}$
 - 2. rotate about the origin of $\{W\}$
 - 3. translate back to the original ICC
- how much rotation over the time interval?
 - angular velocity * elapsed time = $\omega \delta t$

$$\begin{bmatrix} \cos(\omega \delta t) & -\sin(\omega \delta t) \\ \sin(\omega \delta t) & \cos(\omega \delta t) \end{bmatrix} \begin{bmatrix} x - ICC_x \\ y - ICC_y \end{bmatrix}$$

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- computing the rotation about the ICC
 - 1. translate so that the ICC moves to the origin of $\{W\}$
 - 2. rotate about the origin of $\{W\}$
 - 3. translate back to the original ICC

$$\begin{bmatrix} x(t+\delta t) \\ y(t+\delta t) \end{bmatrix} = \begin{bmatrix} \cos(\omega \delta t) & -\sin(\omega \delta t) \\ \sin(\omega \delta t) & \cos(\omega \delta t) \end{bmatrix} \begin{bmatrix} x-ICC_x \\ y-ICC_y \end{bmatrix} + \begin{bmatrix} ICC_x \\ ICC_y \end{bmatrix}$$

L

- what about the orientation $\theta(t+\delta t)$?
 - just add the rotation for the time interval
- new pose

$$\begin{bmatrix} x(t+\delta t) \\ y(t+\delta t) \end{bmatrix} = \begin{bmatrix} \cos(\omega \delta t) & -\sin(\omega \delta t) \\ \sin(\omega \delta t) & \cos(\omega \delta t) \end{bmatrix} \begin{bmatrix} x-ICC_x \\ y-ICC_y \end{bmatrix} + \begin{bmatrix} ICC_x \\ ICC_y \end{bmatrix}$$
$$\theta(t+\delta t) = \theta + \omega \delta t$$

which can be written as

$$\begin{bmatrix} x(t+\delta t) \\ y(t+\delta t) \\ \theta(t+\delta t) \end{bmatrix} = \begin{bmatrix} \cos(\omega \delta t) & -\sin(\omega \delta t) & 0 \\ \sin(\omega \delta t) & \cos(\omega \delta t) & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} x-ICC_x \\ y-ICC_y \\ \theta \end{bmatrix} + \begin{bmatrix} ICC_x \\ ICC_y \\ \omega \delta t \end{bmatrix}$$

- the previous equation is valid if $v_L \neq v_R$
 - i.e., if the differential drive is not travelling in a straight line
- if $v_L = v_R = v$ then

$$\begin{bmatrix} x(t+\delta t) \\ y(t+\delta t) \\ \theta(t+\delta t) \end{bmatrix} = \begin{bmatrix} x+v\delta t\cos\theta \\ y+v\delta t\sin\theta \\ \theta \end{bmatrix}$$



- given the forward kinematics of the differential drive it is easy to write a simulation of the motion
 - we need a way to draw random numbers from a normal distribution
 - in Matlab
 - randn(n) returns an n-by-n matrix containing pseudorandom values drawn from the standard normal distribution
 - see mvnrnd for random values from a multivariate normal distribution

POSE = [];	%	final pose of robot after each trial
sigma = 0.01;	%	noise standard deviation
L = 0.2;	%	distance between wheels
dt = 0.1;	%	time step
TRIALS = $1000;$	%	number of trials

for trial = 1:TRIALS



end



```
theta = pose(3);
        if vr == vl
           pose = pose + [vr * cos(theta) * dt;
                            vr * sin(theta) * dt;
                            0];
        else
           omega = (vr - vl) / L;
to a cot is
           R = (L / 2) * (vr + vl) / (vr - vl);
            ICC = pose + [-R * sin(theta);
                            R * cos(theta);
                            01;
           pose = rz(pomega * dt) * (pose - ICC) + ICC +
                    [0; 0; omega * dt];
                                         - vandom normal variable with mean = 0, \sigma = 1
        end
        vr = 1 + sigma * randn(1);
        vl = 1 + sigma * randn(1);
```